Discussion Paper Series

General Equilibrium Real Exchange Rates in Three-Good Economy Setting

Jacob Oduor

DP/110/2010

THE KENYA INSTITUTE FOR PUBLIC POLICY RESEARCH AND ANALYSIS (KIPPRA)

General Equilibrium Real Exhange Rates in Three-Good Economy Setting

Jacob Oduor

Macroeconomics Division
Kenya Institute for Public Policy
Research and Analysis

KIPPRA Discussion Paper No. 110
2010

KIPPRA in Brief

The Kenya Institute for Public Policy Research and Analysis (KIPPRA) is an autonomous institute whose primary mission is to conduct public policy research leading to policy advice. KIPPRA's mission is to produce consistently high-quality analysis of key issues of public policy and to contribute to the achievement of national long-term development objectives by positively influencing the decision-making process. These goals are met through effective dissemination of recommendations resulting from analysis and by training policy analysts in the public sector. KIPPRA therefore produces a body of well-researched and documented information on public policy, and in the process assists in formulating long-term strategic perspectives. KIPPRA serves as a centralized source from which the Government and the private sector may obtain information and advice on public policy issues.

Published 2010

Bishops Garden Towers, Bishops Road
PO Box 56445, Nairobi, Kenya
tel: +254 20 2719933/4; fax: +254 20 2719951
email: admin@kippra.or.ke
website: http://www.kippra.org

ISBN 9966 777 69 5

The Discussion Paper Series disseminates results and reflections from ongoing research activities of the Institute's programmes. The papers are internally refereed and are disseminated to inform and invoke debate on policy issues. Opinions expressed in the papers are entirely those of the authors and do not necessarily reflect the views of the Institute.

KIPPRA acknowledges generous support from the European Union (EU), the African Capacity Building Foundation (ACBF), the United States Agency for International Development (USAID), the Department for International Development of the United Kingdom (DfID) and the Government of Kenya (GoK).

Abstract

This study develops a theoretical general equilibrium model of the determination of the equilibrium long-run real exchange rates in a three-good open economy framework. Three economic agents are considered in the model: a representative consumer, a representative firm, and the government sector. Two goods are produced by the representative firm: a non-tradable good, and an exportable good while the consumer demands two goods: the non-tradable good, and an imported good. The three agents interact in an internal-external equilibrium framework. Comparative statistics from the developed equilibrium long-run real exchange rate show that the long-run real exchange rate depreciates as government spending on the traded goods increases, and it appreciates as foreign interest rates increase. On the other hand, an increase in the world interest rates appreciates the real exchange rates, while an increase in capital outflow depreciates the real exchange rates. In general, this study adds to the existing knowledge of the determination of the long-run real exchange rates by suggesting that the real sector variables, including the capital labour ratio, are important determinants of the long-run real exchange rates.

Abbreviations and Acronyms

BEER	Behavioural Equilibrium Exchange Rates
FEER	Fundamental Equilibrium Exchange Rate
LRER	Long-Run Equilibrium Real Exchange Rates
NATREX	Natural Real Exchange Rates
PPP	Purchasing Power Parity

Table of Contents

1. Introduction

Several different approaches have been used in the literature to define the Long-Run Equilibrium Real Exchange Rates (LRER). The most common is the Purchasing Power Parity (PPP) approach and its extensions, including the Balassa-Samuelson approach and the Capital Enhanced Measures of the Equilibrium Real Exchange Rates (CHEERS) approach. The other approach is the Permanent and Transitory Decomposition of the Equilibrium Real Exchange Rates (PEER) approach, which includes the Beveridge-Nelson Decompositions, the Structural Vector Autoregression Approach, and the Cointegration Based PEERS. The other approaches are the Behavioural Equilibrium Exchange Rates (BEER) approach and the Internal-External Equilibrium (I-E) approach, which includes the Fundamental Equilibrium Exchange Rate (FEER) approach, and the Natural Real Exchange rates (NATREX) approach. Other than the I-E equilibrium approach, the other approaches are partial equilibrium approaches. No comprehensive approach has been developed to determine a general equilibrium real exchange rate with realistic assumptions. Edwards (1989), for instance, develops a general equilibrium approach to the real exchange rate determination but assumes only one factor of production, labour. This is apparently done to make mathematical calculations easier. The disadvantage of these assumptions is that the resultant equilibrium will not hold in reality, since most countries have freely mobile capital flows and more than one factor of production. It is therefore necessary to develop a theoretical model of the determination of the real exchange rate that takes into account the current dynamics of these economies.

This paper develops a general equilibrium real exchange rate that allows for both real and nominal factors to play a role in the short-run in line with the internal-external equilibrium approach. The starting point in our model development is Edwards (1989), who defines the long-run equilibrium real exchange rate as that rate achieved when the external sector (balance of payments) and the internal sector (non-tradable goods market) are simultaneously in equilibrium. Additionally, the long run equilibrium requires that fiscal policy is sustainable; that is, the government budget is balanced and portfolio equilibrium holds. In the long-run, however, only real factors - the ‘fundamentals’ - influence the equilibrium real exchange rate.

2. Structure of the Model

The model developed in this section considers a three-good framework - exportables, importables, and non-tradables in a small open economy. There is a floating exchange rate regime, and there exists the asset markets and a government sector. It is assumed that this country produces the exportable and non-tradable goods and consumes the importable and the non-tradable goods. Nationals of this country hold both domestic assets and foreign assets. The government consumes both importable goods and non-tradable goods, and uses non-distortionary taxes and domestic debt to finance its expenditures. There is no money in the model. It is assumed that the government, as is the private sector, cannot borrow from abroad. Finally, it is assumed that there is perfect foresight.

2.1 Demand Side

The demand side of the model is constructed with a representative agent maximizing his utility over the consumption of two goods: imports and non-tradable goods.

2.1.1 The households

For simplicity, an overlapping generation representative consumer who lives for two periods is considered. He works when young and consumes when old. He consumes two goods in each period: the non-tradable goods *n* and (the tradable) imported goods *m*. The consumer must consume at least some positive units of each good in each period. His preferences over *n* and *m* in each period is represented by an additively separable utility function. In every period, every member of the household is endowed with one unit of labour, which he supplies inelastically. For this, he receives a wage income of w_t per period and pays tax at the rate $0<tax<1$ to the government. His total after tax labour income in period *t* is therefore $(1-tax)w_t$. Individuals are the same within and across generations. The gross returns to savings enable consumers to consume during the second period of their lives. Non-traded goods prices are taken as the numeraire and are therefore normalized to one and given by $p_t = 1$ in period one and $p_{t+1} = 1$ in the next period. Importable goods prices are given as e_t, which is the relative price of the importable goods in terms of the non-traded goods; that is, the units of the non-tradable goods that would be exchanged for one unit of the imported goods. From this definition, e_t will denote the real exchange rate in the model. This definition is in line with Montiel (1999) who defines the real exchange rate as the relative price of importable goods in terms of the non-tradaded goods. The assumption is that the supply of the imported good *m* from the world market is enough to cater for the demand of *m* in the domestic economy in every period. The consumers' income that is not used in the consumption of the two goods in period one is invested in two assets, either domestic assets (bonds) B^*_t or foreign assets (bonds) B_t. Both assets are one-period bonds. The consumers receive returns on their bond holdings in the next period, which he uses to consume in the second period. He consumes all his savings in the second period.

Assumption 1

The utility function $u : \Re^4_+ \rightarrow \Re$, is differentiable, strictly quasi concave, strictly monotonically increasing, and homothetic.

The consumer's choice behaviour is represented by an additively separable utility function of the form:

$$u\left(n_t, m_t, n_{t+1}, m_{t+1}\right) = \ell n\left(n_t^{\rho} m_t^{1-\rho}\right) + \upsilon \ell n\left(n_{t+1}^{\rho} m_{t+1}^{1-\rho}\right) \tag{2.1}$$

And his budget constraints are given as:

$$\begin{aligned} n_t + e_t m_t + B_t + e_t B_t^* &= (1 - tax)\, w_t \\ n_{t+1} + e_{t+1} m_{t+1} &= (1 + i_t)\, B_t + \left(1 + i^*{}_t\right) e_t B_t^* \end{aligned} \tag{2.2}$$

where $0<\upsilon<1$ is a subjective discount factor (The consumers do not value tomorrow's consumption as much as they value today's consumption). n_t>0, m_t>0 and $0<\rho<1$ is the substitution parameter.

$(1 + i_t)$ is the return on domestic bond, given as the non-tradable goods per unit of domestic bond and $(1 + i_t^*)$ is the return on foreign bond in terms of the domestic non-tradable goods. The returns are determined in period *t*, hence the use of the relative price e_t. The assumption that preferences are monotonic ensures that the budget constraint holds with equality. The consumer's problem is therefore to find non-negative consumption bundles solving:

$$\max u\left(n_t, m_t, n_{t+1}, m_{t+1}\right) = \ell n\left(n_t^{\rho} m_t^{1-\rho}\right) + \upsilon \ell n\left(n_{t+1}^{\rho} m_{t+1}^{1-\rho}\right) + \lambda_1 \begin{pmatrix} (1-tax) w_t - \\ n_t - e_t m_t - B_t - e_t B_t^* \end{pmatrix} +$$
$$\lambda_2 \left((1+i_t) B_t + \left(1+i_t^*\right) e_t B_t^* - n_{t+1} - e_{t+1} m_{t+1}\right) \tag{2.3}$$

The domestic bonds are supplied by the government. The returns from the two investment alternatives (investment in domestic bonds and investment in foreign bonds) are assumed to be equal, hence the two investment alternatives are perfect substitutes, i.e:

$$(1+i_t)=(1+i_t^*) \tag{2.4}$$

In addition, the assumption is that the consumer must hold at least a positive amount of each of the bonds. The assumption that the two bonds are perfect substitutes ensures that the equality in (2.4) holds in all periods. To simplify the problem further;

Let $s_t=\left(B_t/e_t+B_t^*\right)$ where $B>0$ and $B^*>0$ (2.5)

And let $\tilde{i}_{t+1}=(1+i_t)=(1+i_t^*)$

Then, the consumer's utility maximization problem can be written as:

$$\max u(n_t,m_t,n_{t+1},m_{t+1})=\ell n\left(n_t^\rho m_t^{1-\rho}\right)+\upsilon\ell n\left(n_{t+1}^\rho m_{t+1}^{1-\rho}\right) s.t.\begin{cases} n_t+e_tm_t+e_ts_t=(1-tax)w_t \\ n_{t+1}+e_{t+1}m_{t+1}=\tilde{i}_{t+1}(e_ts_t)\end{cases}$$

Solving for m_t and n_t from the budget constraints,

$$m_t=\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}$$

$$m_{t+1}=\frac{\tilde{i}_{t+1}(e_ts_t)-n_{t+1}}{e_{t+1}}$$

The maximization problem then becomes:

$$u(n_t,n_{t+1})=\ell n\left(n_t^\rho\left(\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}\right)^{1-\rho}\right)+\upsilon\ell n\left(n_{t+1}^\rho\left(\frac{\tilde{i}_{t+1}(e_ts_t)-n_{t+1}}{e_{t+1}}\right)^{1-\rho}\right) \tag{2.6}$$

Solving for the first order conditions:

$$\frac{\partial\ell(n_t,n_{t+1})}{\partial n_t}=\left(n_t^\rho\left(\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}\right)^{1-\rho}\right)^{-1}$$
$$\begin{bmatrix}\rho n_t^{\rho-1}\left(\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}\right)^{1-\rho}+ \\ n_t^\rho(1-\rho)\left(\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}\right)^{-\rho}\frac{-1}{e_t}\end{bmatrix}\overset{!}{=}0 \tag{2.7-i}$$

$$\frac{\partial\ell(n_t,n_{t+1})}{\partial n_{t+1}}=\upsilon\left(n_{t+1}^\rho\left(\frac{\tilde{i}_{t+1}(e_ts_t)-n_{t+1}}{e_{t+1}}\right)^{1-\rho}\right)^{-1}$$
$$\begin{bmatrix}\rho n_{t+1}^{\rho-1}\left(\frac{\tilde{i}_{t+1}(e_ts_t)-n_{t+1}}{e_{t+1}}\right)^{1-\rho}+ \\ n_{t+1}^\rho(1-\rho)\left(\frac{\tilde{i}_{t+1}(e_ts_t)-n_{t+1}}{e_{t+1}}\right)^{-\rho}\frac{-1}{e_t}\end{bmatrix}\overset{!}{=}0 \tag{2.7-ii}$$

$$\frac{\partial \ell(n_t,n_{t+1})}{\partial s_t}=\left(n_t^{\rho}\left(\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}\right)^{1-\rho}\right)^{-1}n_t^{\rho}(1-\rho)\left(\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}\right)^{-\rho}-1+$$ (2.7-iii)

$$\upsilon\left(n_{t+1}^{\rho}\left(\frac{\tilde{i}_{t+1}(e_ts_t)-n_{t+1}}{e_{t+1}}\right)^{1-\rho}\right)^{-1}n_{t+1}^{\rho}(1-\rho)\left(\frac{\tilde{i}_{t+1}(e_ts_t)-n_{t+1}}{e_{t+1}}\right)^{-\rho}\frac{e_t\tilde{i}_{t+1}}{e_{t+1}}\overset{!}{=}0$$

Because the utility function is strictly concave and strictly monotonically increasing, the contemporary slackness conditions hold.

From condition (2.7-i)

$$\frac{\rho n_t^{\rho-1}\left(\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}\right)^{1-\rho}+n_t^{\rho}(1-\rho)\left(\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}\right)^{-\rho}-\frac{-1}{e_t}}{n_t^{\rho}\left(\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}\right)^{1-\rho}}\overset{!}{=}$$

$$\Leftrightarrow \rho n_t^{\rho-1}\left(\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}\right)^{1-\rho}+n_t^{\rho}(1-\rho)\left(\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}\right)^{-\rho}\frac{-1}{e_t}=0$$

$$\Leftrightarrow \rho n_t^{\rho-1}\left(\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}\right)^{1-\rho}+n_t^{\rho}(1-\rho)\left(\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}\right)^{-\rho}\frac{-1}{e_t}=0$$

$$\Leftrightarrow \rho\left((1-tax)w_t-e_ts_t\right)=n_t(1-\rho)+\rho n_t$$ (2.8)

$$\Leftrightarrow n_t=\rho\left((1-tax)w_t-e_ts_t\right)$$

and $m_t(e_t,w_t)=(1-\rho)\dfrac{(1-tax)w_t-e_ts_t}{e_t}$ (2.9)

From condition (2.7-ii)

$$\left[\rho n_{t+1}^{\rho-1}\left(\frac{\tilde{i}_{t+1}(e_ts_t)-n_{t+1}}{e_{t+1}}\right)^{1-\rho}\right]+n_{t+1}^{\rho}(1-\rho)\left(\frac{\tilde{i}_{t+1}(e_ts_t)-n_{t+1}}{e_{t+1}}\right)^{-\rho}\frac{-1}{e_{t+1}}=0$$

$$\rho\frac{\tilde{i}_{t+1}(e_ts_t)-n_{t+1}}{e_{t+1}}=(1-\rho)\frac{n_{t+1}}{e_{t+1}}+\rho\frac{n_{t+1}}{e_{t+1}}$$

$$\rho\frac{\tilde{i}_{t+1}(e_ts_t)}{e_{t+1}}=\frac{n_{t+1}}{e_{t+1}}\left((1-\rho+\rho)\right)$$

$$\Rightarrow n_{t+1}=\rho\tilde{i}_{t+1}(e_ts_t)$$ (2.10)

$$\Rightarrow m_{t+1}=(1-\rho)\frac{\tilde{i}_{t+1}(e_ts_t)}{e_{t+1}}$$ (2.11)

From (2.7-iii)

$$\Leftrightarrow\left(n_t^{\rho}\left(\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}\right)^{1-\rho}\right)^{-1}n_t^{\rho}(1-\rho)\left(\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}\right)^{-\rho}(-1)+$$

$$\upsilon\left(n_{t+1}^{\rho}\left(\frac{\tilde{i}_{t+1}(e_ts_t)-n_{t+1}}{e_{t+1}}\right)^{1-\rho}\right)^{-1}n_{t+1}^{\rho}(1-\rho)\left(\frac{\tilde{i}_{t+1}(e_ts_t)-n_{t+1}}{e_{t+1}}\right)^{-\rho}\frac{e_t\tilde{i}_{t+1}}{e_{t+1}}\overset{!}{=}0$$

$$\Leftrightarrow\left(\frac{(1-tax)w_t-e_ts_t-n_t}{e_t}\right)^{-1}(1-\rho)(-1)+\upsilon\left(\frac{\tilde{i}_{t+1}(e_ts_t)-n_{t+1}}{e_{t+1}}\right)^{-1}(1-\rho)\frac{e_t\tilde{i}_{t+1}}{e_{t+1}}\overset{!}{=}0$$

$$\Leftrightarrow\left(\frac{(1-\rho)(1-tax)w_t-e_ts_t}{e_t}\right)^{-1}(-1)+\upsilon\left(\frac{(1-\rho)\tilde{i}_{t+1}(e_ts_t)}{e_{t+1}}\right)^{-1}\frac{e_t\tilde{i}_{t+1}}{e_{t+1}}\overset{!}{=}0$$

$$\Leftrightarrow-\left(\frac{(1-\rho)(1-tax)w_t-e_ts_t}{e_t}\right)^{-1}+\upsilon\left((1-\rho)\tilde{i}_{t+1}(e_ts_t)\right)^{-1}e_t\tilde{i}_{t+1}\overset{!}{=}0$$

$$\Leftrightarrow\left(\frac{(1-tax)w_t-e_ts_t}{e_.}\right)^{-1}=\upsilon\left(\tilde{i}_{t+1}(e_ts_t)\right)^{-1}e_t\tilde{i}_{t+1}$$

$$\Leftrightarrow\frac{\upsilon((1-tax)w_t-e_ts_t)}{e_t}=s_t$$

$$\Leftrightarrow s_t=\frac{\upsilon}{e_t(1+\upsilon)}(1-tax)w_t \tag{2.12}$$

2.1.3 Capital stock

The output of the export good forms new capital stock used by the firm in production in the next period. Capital depreciates fully at the end of period *t*; therefore the capital stock in period $t+1$ is equal to the output of the export good, i.e.

$$\Leftrightarrow k_{t+1}=F_x(K_t^x L_t^x)=l_t^x f_x\left(k_t^x\right) \tag{2.13}$$

2.1.4 Government

In period *t* the government consumes two goods: the non-tradable good $g_t^n \geq 0$ and the imported good $g_t^m \geq 0$. The total government expenditure in period *t* is $g_t = e_t g_t^m + g_t^n$. The choice between tradable good and non-tradable good is assumed to be exogenous. If the total household before-tax-income in period *t* is denoted by w_t and $0 \leq tax \leq 1$ is the amount of proportional tax rate on the consumer's income, then the government receives $tax(w_t)$ as revenue from taxes. The government budget is balanced when:

$$\left(g_t^n+e_tg_t^m\right)=(tax)w_t \tag{2.14}$$

If its spending is greater than its tax revenue, then the government runs a budget deficit. The government finances its deficits using domestic debt. This is done by issuing one-period domestic bonds B_t at the domestic rates of interest i_t in period *t*. The government primary budget deficit is thus given as:

$$Def_t=\left(g_t^n+e_tg_t^m\right)+\left(1+i_{t-1}\right)B_{t-1}-(tax)w_t-B_t \tag{2.15}$$

The total bond supply in period t is given by:

$$B_t = \left(g_t^{\,n} + e_t g_t^{\,m}\right) + \left(1 + i_{t-1}\right) B_{t-1} - \left(tax\right) w_t \tag{2.16}$$

where B_t is the domestic assets issued by the government (constituting domestic debt) and i_t is the domestic rate of interest.

2.1.5 Market clearing condition in the non-tradable goods market

The non-tradable good is demanded by the households in the amount $n_t^{\,d}$ and the government demands $g_t^{\,n}$. The non-tradable good is supplied by the firms in the amount of $n_t^{\,s}$. The market clearing condition in the non-tradable good market therefore requires that:

$$n_t^{\,d} + g_t^{\,n} = n_t^{\,s} \tag{2.17}$$

Montiel (1999) refers to the e_t that solves equation (2.17) as the short-run equilibrium real exchange rate in the sense that it clears the market for the non-tradable good for given values of private consumption. They note that this real exchange rate will be sustainable only to the extent that private consumption is itself sustainable.

2.2 Supply Side

On the supply side, we assume a representative firm, producing two different goods, non-tradable goods n_t at prices $p_t = 1$ in period *t* (the domestic prices are normalized to one) and the exportable goods x_t at prices $\tilde{p}_t$ in period *t* (therefore $\tilde{p}_t$ is the relative price of the exportable goods x_t in terms of the non-tradable good n_t). The firm uses constant returns to scale technology under perfect competition in its production plans. The non-tradable good is used solely for domestic consumption, while the exportable good is produced for exports and is not consumed domestically. We assume that there is enough demand for the exportable good x in the other countries to absorb all the supply in every period since ours is a small country. Good x is the capital good and is thus used to generate next period's capital. The firms produce the two goods using variable units of labour and capital. We also assume that there are no inventories at the end of each period. The firm's objective is to minimize costs and maximize profits.

2.2.1 The firm's cost minimization problem

The firm's problem in every period is to find a wage-rental combination that would produce the optimal level of output n_t and x_t at minimum cost. The same wage rental ratio that buys this input combination is the same wage rental ratio that maximizes the firm's profits.

Assumption 2

Both production functions $f_i : \Re^2_+ \longrightarrow \Re$ are twice continuously differentiable, strictly increasing and strictly concave, i.e. $f_i''(k_t)<0$ and $f_i'(k_t)>0$, $\forall\ k>0$, $i = n_t, m_t$ and $k_t := \frac{K_t}{\gamma_t L_t}$ where γ_t is the labour-augmenting technical progress.

There are two factors of production, L_t and K_t such that the production functions used to produce good n_t in extensive form is of the form:

$$F_n\left(L^n{}_t, K^n{}_t\right) = \frac{\alpha}{\beta} K^n{}_t{}^{\beta} \gamma_t L^n{}_t{}^{1-\beta} \geq n_t \tag{2.18}$$

The firm's problem in sector n is to minimize costs

$\min\limits_{(K^n, L^n \geq 0)} c_t = w_t \gamma_t L^n_t + r_t K^n_t$ with $w_t>0$ and $r_t>0$ s.t $F_n\left(L^n_t, K^n_t\right) = \frac{\alpha}{\beta} K_t^{n\beta} \gamma_t L_t^{n1-\beta} \geq n_t$

The firm therefore wishes to minimize its cost on the two inputs

$$\min_{(K^n, L^n \geq 0)} c_t\left(n_t, w_t, r_t\right) := \left\{ w_t \gamma_t L^n_t + r_t K^n_t \,\middle|\, \frac{\alpha}{\beta} K_t^{n\beta} \gamma_t L_t^{n1-\beta} \geq n_t \right\} \tag{2.19}$$

Because the production technology is strictly monotonic, we can write the constraint with equality, hence:

$$\min_{(K^n, L^n \geq 0)} c_t \left\{ w_t \gamma_t L^n + r_t K^n \,\middle|\, \frac{\alpha}{\beta} K^n{}_t{}^{\beta} \gamma_t L^n{}_t{}^{1-\beta} = n_t \right\} \tag{2.20}$$

Forming the Lagrangian:

$$\ell(K_t^n, L_t^n) = \min_{(K_t^n, L_t^n \geq 0)} c_t \left\{ w_t \gamma_t L_t{}^n + r_t K_t{}^n - \lambda\left(n_t - \frac{\alpha}{\beta} K_t^{n\beta} \gamma_t L_t^{n1-\beta} \right)\right\}$$

The first order conditions are given as:

$$\partial \ell(K_t^n, L_t^n) \big/ \partial K_t^n = r_t + \lambda \alpha K_t{}^{\beta-1} \gamma_t L_t{}^{1-\beta} \overset{!}{=} 0 \tag{2.21}$$

$$\partial \ell(K_t, L_t) \big/ \partial L_t = w_t + \lambda \frac{\alpha(1-\beta)}{(\beta)} \gamma_t L_t{}^{-\beta} K_t{}^{\beta} \overset{!}{=} 0 \tag{2.22}$$

Taking the ratio of the two first order conditions,

$$\frac{r_t}{w_t} = \frac{\lambda \alpha K_t^{\beta-1} \gamma_t L_t^{1-\beta}}{\lambda \frac{\alpha(1-\beta)}{(\beta)} \gamma_t L_t^{-\beta} K_t^{\beta}} \Rightarrow \frac{r_t}{w_t} = \frac{\beta}{1-\beta} K_t^{\beta-1} K_t^{-\beta} L_t^{1-\beta} L_t^{\beta}$$

$$\Rightarrow \frac{r_t}{w_t} = \frac{\beta}{1-\beta} \frac{L_t}{K_t} \quad \Rightarrow \frac{w_t}{r_t} = \frac{1-\beta}{\beta} k_t^n \text{ where } k_t := \frac{K_t}{L_t}$$

Let us define $\omega := \frac{w_t}{r_t}$, then

$$\Rightarrow \omega\left(k_t^n\right) := \frac{w_t}{r_t} = \frac{1-\beta}{\beta} k_t^n \tag{2.23}$$

Similarly, $\omega_t\left(k^x\right) := \frac{w_t}{r_t} = k^x \frac{1-\delta}{\delta}$ (2.24)

$\omega(k_t)$ is the cost minimizing wage-rental ratio. We will see later that this cost minimizing wage-rental ratio is the same profit maximizing wage-rental ratio.

2.2.2 Production

In the production of the two goods, we adopt the two-sector production model in line with Galor (1992). This section has also benefited from discussions of the two-sector models in Schmitz (2007) and Teodorescu (2008). The firm's objective is to minimize costs and maximize profits. The total output of the non-tradable good is given as:

$$N_t = F_n\left(K_t^n, \gamma_t L_t^n\right)$$

where L^n and K^n denote the amounts of labour and capital employed in the non-tradable sector. Since the production function F_n is linear homogenous, it can be re-written as:

$$\gamma_t L_t^n F_n\left(K_t^n / \gamma_t L_t^n , 1\right) =: \gamma_t L_t^n f_n\left(k_t^n\right) \tag{2.25}$$

If we define $k_t^j := \frac{K_t^j}{\gamma_t L_t^j}$ as the capital labour ratio in sector j at time t , $j = n, x$ then by equation (2.25), the per capita output of the non-tradable good in period t is given by:

$$n_t := N_t / \gamma_t L_t = l_t^n f_n\left(k_t^n\right) \tag{2.26}$$

where $l_t^n = L_t^n / \gamma_t L_t$

Similarly, the traded good is produced using capital and labour in every period using a time-invariant constant return to scale production technology. The total output of the exportable good is given as:

$$X_t = F_x\left(K_t^x, \gamma_t L_t^x\right) = L_t^x F_x\left(K_t^x / \gamma_t L_t^x , 1\right) =: \gamma_t L_t^x f_x\left(k_t^x\right) \tag{2.27}$$

Defining the per capita output of the exportable good as $x_t := X_t / \gamma_t L_t$, then by equation (2.27), we obtain the per capita output of the exportable good in period t as:

$$x_t \equiv \frac{X_t}{\gamma_t L_t} = l_t^x f_x\left(k_t^x\right) \tag{2.28}$$

where $l_t^x = L_t^x / L_t$

Let the production functions in extensive form be given as:

$$F_n\left(\gamma_t L^n{}_t, K^n{}_t\right) = \frac{\alpha}{\beta} K^n{}_t{}^{\beta} \gamma_t L^n{}_t{}^{1-\beta} \tag{2.29}$$

for the non-traded goods production, and

$$F_x\left(\gamma_t L_t^x, K_t^x\right) = \frac{\varphi}{\delta} K_t^{x\delta} \gamma_t L_t^{x1-\delta} \tag{2.30}$$

for the traded (exportable) goods production technology. The production functions in intensive form are given as:

$$f_n\left(k_t^n\right) = \frac{\alpha}{\beta}\left(k_t^n\right)^{\beta} \quad , \; k_t^n > 0 \; , \; \alpha > 0 \; , \; \beta \in (0,1) \tag{2.31}$$

$$f_x\left(k_t^x\right) = \frac{\varphi}{\delta}\left(k_t^x\right)^{\delta} \; , \; k_t^x > 0 \; , \; \varphi > 0 \; , \; \delta \in (0,1) \tag{2.32}$$

We assume that both production sectors are profit maximizers. Each sector will therefore choose an input combination that will maximize its profits. Since the firm operates under perfect competition, it has no influence on either the prices of goods or input prices. The firm's profit maximization problem in the non-traded good sector is therefore given by:

$$\max_{K_t^n, L_t^n} \left\{F_n\left(K_t^n, \gamma_t L_t^n\right) - r_t K_t^n + w_t \gamma_t L_t^n\right\} \tag{2.33}$$

where r_t is the return to capital at time t and w_t is the wage rate at time t.

Since the production function exhibits constant returns to scale, the profit maximization problem in the non-traded goods sector can be written in the form:

$$\max_{K^n, L^n} \left\{\gamma_t L^n f_n\left(K^n \big/ \gamma_t L^n\right) - r_t K^n + w_t \gamma_t L^n\right\}$$

The first order conditions with respect to K^x and L^x are then given as;

$$f_n'(k_n) - r_t = 0 \tag{2.34}$$

$$f_n\left(k_t^n\right) - k_t^n f_n'\left(k_t^n\right) - w_t = 0 \tag{2.35}$$

where $k_t^n = \dfrac{K_t^n}{\gamma_t L_t^n}$ (2.35a)

These first order conditions imply that:

$$r_t = f_n'(k_n) \tag{2.36}$$

$$w_t = f_n\left(k_t^n\right) - k_t^n f_n'\left(k_t^n\right) \tag{2.37}$$

Similarly, the firm's profit maximization problem in the traded (exportable) good sector is given by:

$$\max_{K^x, L^x} \left\{\tilde{p}_t F_x\left(K^x, \gamma_t L^x\right) - rK^x + w\gamma_t L^x\right\} \tag{2.38}$$

The first order conditions with respect to K^n and L^n are then given as:

$$\tilde{p}_t f_x'(k_x) - r_t = 0 \tag{2.39}$$

$$\tilde{p}_t\left[f_x(k_t^x) - k_t^x f_x'(k_t^x)\right] - w_t = 0 \tag{2.40}$$

where $k_t^x \equiv \frac{K_t^x}{\gamma_t L_t^x}$

This implies that:

$$r_t = \tilde{p}_t f_x'(k_x) \tag{2.41}$$

$$w_t = \tilde{p}_t\left[f_x(k_t^x) - k_t^x f_x'(k_t^x)\right] \tag{2.42}$$

Both goods and factor markets are perfectly competitive, meaning that:

$$r_t = \tilde{p}_t f_x'(k_t^x) = f_n'(k_t^n) \tag{2.43}$$

$$w_t = \tilde{p}_t\left[f_x(k_t^x) - k_t^x f_x'(k_t^x)\right] = f_n(k_t^n) - k_t^n f_n'(k_t^n) \tag{2.44}$$

2.2.3 Market clearing conditions in the factor markets

We assume that factors are fully employed in both sectors in period *t*. In period *t*, total labour L_t is supplied inelastically by the households while the total capital stock K_t is supplied inelastically by the foreigners. The total demand for capital $K_t^n + K_t^x$ and the total demand for labour $L_t^n + L_t^x$ are described by the first order conditions of the profit maximization. Therefore:

$$K_t = K_t^n + K_t^x$$

But $K_t^j = k_t^j \gamma_t L_t^j$ (see equation 2.35a)

$$\Rightarrow \frac{k_t^n \gamma_t L_t^n}{\gamma_t L_t} + \frac{k_t^x \gamma_t L_t^x}{\gamma_t L_t} = \frac{K_t}{\gamma_t L_t}$$

Since $\frac{L_t^n}{L_t} = l_t^n$, $\frac{L_t^x}{L_t} = l_t^x$ and $\frac{K_t}{\gamma_t L_t} = k_t$ then full employment of capital imply that:

$$K_t^n + K_t^x = K_t \Leftrightarrow k_t^n l_t^n + k_t^x l_t^x = k_t \tag{2.45}$$

Full employment of labour, on the other hand, is given by $L_t^n + L_t^x = L_t$
Since $l_t^j = L_t^j / \gamma_t L_t$ describes the proportion of labour force employed in sector j in period t, for $j = n, x$,

$$L_t^n + L_t^x = L_t \quad \Leftrightarrow \quad l_t^n + l_t^x = 1 \tag{2.46}$$

2.45 $\Rightarrow l^n = \dfrac{k_t - k^x}{k^n - k^x}$

2.46 $\Rightarrow l^x = \dfrac{k_t - k^n}{k^x - k^n}$

These are the market clearing conditions on the two factor markets.

2.2.4 Profit maximizing wage-rental ratio

We define the ratio of wage to the rate of interest evaluated at the profit maximizing capital labour ratio as the wage-rental ratio given as:

$$\omega_t := \frac{w_t}{r_t} = \frac{f_j\left(k_t^j\right) - k_t^j f_j'\left(k_t^j\right)}{f_j'\left(k_t^j\right)}\ ,\ j = n, x \tag{2.47}$$

$$\Rightarrow \frac{f_j\left(k_t^j\right)}{f_j'\left(k_t^j\right)} - k_t^j =: \omega^j\left(k_t^j\right) \text{ for } k_t^j > 0 \tag{2.48}$$

$$\omega^{j'}\left(k_t^j\right) = \frac{f_j\left(k_t^j\right) f''\left(k_t^j\right)}{\left[f_j'\left(k_t^j\right)\right]^2} > 0 \text{ for } k_t^j > 0 \text{ and } j = n, x \tag{2.49}$$

The function $\omega^j\left(k_t^j\right)$ is well defined and, therefore, the inverse function of ω^j is given by k^j, with $w_t = \tilde{p}_t\left[f_x\left(k_t^x\right) - k_t^x f_x'\left(k_t^x\right)\right]$ for $k_t > 0$. $k_t^j\left(\omega_t^j\right)$ is the profit maximizing wage-rental ratio for $k > 0$ and appropriately determined market prices. The market prices are determined as:

$$\tilde{p}_t = \frac{r_t}{f_x'\left(k_t^x\right)} = \frac{f_n'\left(k_n\right)}{f_x'\left(k_t^x\right)} \tag{2.50}$$

From our production functions,

$$k_t^n = \left(\frac{\beta}{1-\beta}\right)\frac{w_t}{r_t} \tag{2.51}$$

$$k_t^x = \left(\frac{\delta}{1-\delta}\right)\frac{w_t}{r_t} \tag{2.52}$$

The capital-labour ratio in sector j at time t is given by:

$$\omega_t\left(k^n\right) := \frac{w_t}{r_t} = k^n\frac{1-\beta}{\beta} \Longrightarrow k^n\left(\omega_t\right) = \omega_t\frac{\beta}{1-\beta} \tag{2.53}$$

$$\omega_t\left(k^x\right) := \frac{w_t}{r_t} = k^x\frac{1-\delta}{\delta} \Longrightarrow k^x\left(\omega_t\right) = \omega_t\frac{\delta}{1-\delta} \tag{2.54}$$

These are the same wage-rental ratios as was derived in the cost minimization problem in section 2.3.1.

It is important to note at this stage that not all positive wage rental ratios are profit maximizing. The set of possible temporary equilibrium wage rental ratios can be reduced to a bounded interval dependent on the total capital labour ratio (Galor, 1992). That is, $\omega^* \in [\omega_{\min}(k_t), \omega_{\max}(k_t)]$ where ω^* is the profit maximizing wage rental ratio. Let us define the quantities $\omega_{\min}(k_t)$ and $\omega_{\max}(k_t)$ by:

$$\omega_{\min}(k_t) := \min\{\omega^n(k_t), \omega^x(k_t)\} \text{ for } k_t > 0 \text{ and} \tag{2.55}$$

$$\omega_{\max}(k_t) := \max\{\omega^n(k_t), \omega^x(k_t)\} \text{ for } k_t > 0 \tag{2.56}$$

Assumption 3

No factor intensity reversal

Either $k_t^n(\omega) > k_t^x(\omega)$ for $\forall\, \omega_t > 0$ or $k_t^x(\omega) > k_t^n(\omega)$ $\forall\, \omega_t > 0$

Assumption 2 implies that, for all feasible wage-rental ratios, either the non-tradable goods sector is more capital-intensive than the traded goods sector or the traded goods sector is more capital-intensive. With this assumption, the $\omega^* \in [\omega_{\min}(k_t), \omega_{\max}(k_t)]$ can be defined as:

$$\omega^* \in [\omega_{\min}(k_t), \omega_{\max}(k_t)] = \begin{cases} [\omega^n(k_t), \omega^x(k_t)] & \text{if } k_t^n(\omega) > k_t^x(\omega) \quad \forall\, \omega_t > 0 \\ [\omega^x(k_t), \omega^n(k_t)] & \text{if } k_t^x(\omega) > k_t^n(\omega) \quad \forall\, \omega_t > 0 \end{cases}$$

If we assume that the traded goods sector is more capital-intensive than the non-traded goods sector, then the capital-intensity condition implies that:

$$k^x(\omega_t) > k^n(\omega_t) \Longrightarrow \delta(1-\beta) > \beta(1-\delta) \Longrightarrow \delta > \beta$$

For k_t exogenously given and $k^x(\omega_t) > k^n(\omega_t)$, then

$$\omega_{\min}(k_t) := \omega^x(k_t) = k_t \frac{1-\delta}{\delta} \text{ and} \tag{2.57}$$

$$\omega_{\max}(k_t) := \omega^n(k_t) = k_t \frac{1-\beta}{\beta} \tag{2.58}$$

From the definition of p_t, as long as both goods are produced, it follows that the prices in period *t* are given by:

$$\tilde{p}_t = \frac{f_n'(k_n(\omega))}{f_x'(k_t^x(\omega))}$$

From our production function:

$$\tilde{p}_t(\omega_t) = \frac{\alpha}{\varphi}\left(\frac{\beta}{1-\beta}\right)^{\beta-1}\left(\frac{\delta}{1-\delta}\right)^{1-\delta}\omega_t^{\beta-\delta} \;\forall\, \omega_t \in [\omega_{\min}(k_t), \omega_{\max}(k_t)] \tag{2.60}$$

where $\omega_t \in (\omega_{\min}(k_t), \omega_{\max}(k_t)) = (\omega^x(k_t), \omega^n(k_t))$ for $k^x(\omega_t) > k^n(\omega_t)$

$\tilde{p}_t(\omega_t)$ is strictly monotonic and, therefore, invertible.

$$\omega_t(\tilde{p}_t) = \left[\tilde{p}_t \frac{\varphi}{\alpha}\left(\frac{\beta}{1-\beta}\right)^{1-\beta}\left(\frac{\delta}{1-\delta}\right)^{\delta-1}\right]^{\frac{1}{\beta-\delta}}$$

Let $\theta := \frac{\varphi}{\alpha}\left(\frac{\beta}{1-\beta}\right)^{1-\beta}\left(\frac{\delta}{1-\delta}\right)^{\delta-1}$. Then, $\omega_t(\tilde{p}_t) = [\tilde{p}_t\theta]^{\frac{1}{\beta-\delta}}$

There exists a single valued function $\omega: [\tilde{p}_{\min}(k_t), \tilde{p}_{\max}(k_t)] \rightarrow [\omega_{\min}(k_t), \omega_{\max}(k_t)]$ such that $\omega_t = \omega(\tilde{p}_t)$.

$$\omega_t = \omega(\tilde{p}_t) = (\tilde{p}_t\theta)^{\frac{1}{\beta-\delta}}$$
$$\Longrightarrow \tilde{p}_t(\omega) = \frac{1}{\theta}\omega_t^{\beta-\delta}$$

Since the profit maximizing wage rental ratio is in the interval $\omega \in [\omega_{\min}(k_t), \omega_{\max}(k_t)]$, and the price $\tilde{p}_t$ is a function of the ω_t, then the profit maximizing prices must also be in the interval $\tilde{p}_t \in (\tilde{p}_{\min}(k_t), \tilde{p}_{\max}(k_t))$ for which $\omega_t \in (\omega_{\min}(k_t), \omega_{\max}(k_t))$.

$$\tilde{p}_t \in [\tilde{p}_{\min}(k_t), \tilde{p}_{\max}(k_t)] = \begin{cases} [\tilde{p}(\omega_{\max}(k_t)), \tilde{p}(\omega_{\min}(k_t))] & if \quad k_t^n(\omega) > k_t^x(\omega) \quad \forall \omega_t > 0 \\ [\tilde{p}(\omega_{\min}(k_t)), \tilde{p}(\omega_{\max}(k_t))] & if \quad k_t^x(\omega) > k_t^n(\omega) \quad \forall \omega_t > 0 \end{cases}$$

For $k^x(\omega_t) > k^n(\omega_t)$, then

$$\omega_{\min}(k_t) = k_t\frac{1-\delta}{\delta} \Rightarrow \tilde{p}_t(\omega_{\min}(k_t)) = \frac{1}{\theta}\left(k_t\frac{1-\delta}{\delta}\right)^{\beta-\delta} \tag{2.65}$$

$$\omega_{\max}(k_t) = k_t\frac{1-\beta}{\beta} \Rightarrow \tilde{p}_t(\omega_{\max}(k_t)) = \frac{1}{\theta}\left(k_t\frac{1-\beta}{\beta}\right)^{\beta-\delta} \tag{2.66}$$

There could exist three situations for:

$$\begin{aligned} &\text{(i)- } n_t = 0 \text{ and } x_t > 0 \quad \text{if } \tilde{p}_t \in [0, \tilde{p}_{\min}(k_t)], \\ &\text{(ii)- } n_t > 0 \text{ and } x_t > 0 \quad \text{if } \tilde{p}_t \in [\tilde{p}_{\min}(k_t), \tilde{p}_{\max}(k_t)], \\ &\text{(iii)- } n_t > 0 \text{ and } x_t = 0 \quad \text{if } \tilde{p}_t \in [\tilde{p}_{\min}(k_t), \infty]; \end{aligned} \tag{2.67}$$

We will continue to assume in the rest of this analysis that the firm produces positive amounts of both goods and, therefore, we will ignore analysis of cases (i) and (iii). For detailed analysis of these situations, see Schmitz (2007) or Galor (1992).

Thus, given the prices $\tilde{p}_t$ at time *t*, the capital-labour ratios in both sectors k_t^n and k_t^x, the wage rate w_t, the interest rate r_t and the wage-rental ratio ω_t are uniquely determined.

$$\forall \tilde{p}_t \in [\tilde{p}_{\min}(k_t), \tilde{p}_{\max}(k_t)],$$
$$r_t = f_n'(k^n(\omega(\tilde{p}_t))) = r(\tilde{p}_t) \tag{2.68}$$

$$w_t = f_n\left(k_t^n\right) - k_t^n f_n'\left(k_t^n\right)$$

$$w_t = f_n\left(k_t^n\left(\omega\left(\tilde{p}_t\right)\right)\right) - k_t^n\left(\omega\left(\tilde{p}_t\right)\right) f_n'\left(k_t^n\left(\omega\left(\tilde{p}_t\right)\right)\right) = w_t\left(\tilde{p}_t\right)$$

The temporary equilibrium capital-labour ratios k_t^n, k_t^x and labour ratios l_t^n, l_t^x.

$$k^n\left(\omega_t\right) = \omega_t \frac{\beta}{1-\beta} \quad \Rightarrow \quad k^n\left(\omega\left(p_t\right)\right) = \frac{\beta}{1-\beta}\left(\tilde{p}_t\theta\right)^{\frac{1}{\beta-\delta}}$$
$$k^x\left(\omega_t\right) = \omega_t \frac{\delta}{1-\delta} \quad \Rightarrow \quad k^x\left(\omega\left(p_t\right)\right) = \frac{\delta}{1-\delta}\left(\tilde{p}_t\theta\right)^{\frac{1}{\beta-\delta}} \tag{2.71}$$

$$l^n\left(k_t, p_t\right) = \frac{k_t - k^x\left(\omega\left(p_t\right)\right)}{k^n\left(\omega\left(p_t\right)\right) - k^x\left(\omega\left(p_t\right)\right)} = \frac{k_t - \frac{\delta}{1-\delta}\left(\tilde{p}_t\theta\right)^{\frac{1}{\beta-\delta}}}{\left(\frac{\beta}{1-\beta} - \frac{\delta}{1-\delta}\right)\left(\tilde{p}_t\theta\right)^{\frac{1}{\beta-\delta}}} \tag{2.72}$$

$$l^x\left(k_t, p_t\right) = \frac{k_t - k^n\left(\omega\left(p_t\right)\right)}{k^x\left(\omega\left(p_t\right)\right) - k^n\left(\omega\left(p_t\right)\right)} = \frac{k_t - \frac{\beta}{1-\beta}\left(\tilde{p}_t\theta\right)^{\frac{1}{\beta-\delta}}}{\left(\frac{\delta}{1-\delta} - \frac{\beta}{1-\beta}\right)\left(\tilde{p}_t\theta\right)^{\frac{1}{\beta-\delta}}} \tag{2.73}$$

And the temporary equilibrium prices on factor markets:

$$r_t = f_n'\left(k^n\left(\omega\left(p_t\right)\right)\right) = \alpha\left(\frac{\beta}{1-\beta}\right)^{\beta-1}\left(\tilde{p}_t\theta\right)^{(\beta-1)/(\delta-\beta)} \tag{2.74}$$

Since $k^n\left(\omega\left(p_t\right)\right) = \frac{\beta}{1-\beta}\left(\tilde{p}_t\theta\right)^{\frac{1}{\beta-\delta}}$ and $f_n' = \beta k_t^{n\,\beta-1}$ (2.75)

$$w_t = f_n\left(k_t^n\left(\omega\left(\tilde{p}_t\right)\right)\right) - k_t^n\left(\omega\left(\tilde{p}_t\right)\right) f_n'\left(k_t^n\left(\omega\left(\tilde{p}_t\right)\right)\right) = w\left(\tilde{p}_t\right) \tag{2.76}$$

$$w\left(\tilde{p}_t\right) = \frac{\alpha}{\beta}\left(\frac{\beta}{1-\beta}\left(\tilde{p}_t\theta\right)^{\frac{1}{\beta-\delta}}\right)^{\beta} - \frac{\beta}{1-\beta}\left(\tilde{p}_t\theta\right)^{\frac{1}{\beta-\delta}}\beta\left(\frac{\beta}{1-\beta}\left(\tilde{p}_t\theta\right)^{\frac{1}{\beta-\delta}}\right)^{\beta-1}$$

$$w\left(\tilde{p}_t\right) = \left(\frac{\alpha}{\beta} - \beta\right)\left(\frac{\beta}{1-\beta}\left(\tilde{p}_t\theta\right)^{\frac{1}{\beta-\delta}}\right)^{\beta}$$

Further, given the per worker capital stock k_t at time *t*, and prices p_t at time *t*, the per worker production of the non-traded good $n_t = f_n\left(k_t^n\right) l_t^n$ and the per worker production of the traded good $x_t = f_x\left(k_t^x\right) l_t^x$ are uniquely determined.

$$n_t = n\left(k_t, e_t\right)$$

$$x_t = x\left(k_t, p_t\right)$$

The temporary equilibrium output of the export good, x is given by:

$$x\left(k_t, p_t\right) = l_t^x f_x\left(k^x\left(\omega\left(p_t\right)\right)\right) \tag{2.77}$$

$$\Rightarrow x(k_t,p_t)=\frac{k_t-\frac{\beta}{1-\beta}(\tilde{p}_t\theta)^{\frac{1}{\beta-\delta}}}{\left(\frac{\delta}{1-\delta}-\frac{\beta}{1-\beta}\right)(\tilde{p}_t\theta)^{\frac{1}{\beta-\delta}}}\frac{\varphi}{\delta}\left(\frac{\delta}{1-\delta}(\tilde{p}_t\theta)^{\frac{\delta}{\beta-\delta}}\right)^{\delta}$$

$$\Rightarrow x(k_t,p_t)=\frac{\left(1-\delta(1-\beta)k_t(\tilde{p}_t\theta)^{-\frac{1}{\beta-\delta}}-\beta(1-\delta)\right)}{\left(\delta(1-\beta)-\beta(1-\delta)\right)}\frac{\varphi}{\delta}\left(\frac{\delta}{1-\delta}(\tilde{p}_t\theta)^{\frac{\delta}{\beta-\delta}}\right)^{\delta}$$

$$\Rightarrow x(k_t,p_t)=\varphi\frac{1-\delta(1-\beta)}{\delta\left(\delta(1-\beta)-\beta(1-\delta)\right)}\left(\frac{\delta}{1-\delta}\right)^{\delta}k_t(\tilde{p}_t\theta)^{\frac{\delta-1}{\beta-\delta}}- \tag{2.78}$$

$$\Rightarrow x(k_t,p_t)=\varphi\frac{1-\delta(1-\beta)}{\delta\left(\delta(1-\beta)-\beta(1-\delta)\right)}\left(\frac{\delta}{1-\delta}\right)^{\delta}k_t(\tilde{p}_t\theta)^{\frac{\delta-1}{\beta-\delta}}-$$

$$\frac{\varphi\left(\beta(1-\delta)\right)}{\delta\left(\delta(1-\beta)-\beta(1-\delta)\right)}\left(\frac{\delta}{1-\delta}\right)^{\delta}(\tilde{p}_t\theta)^{\frac{\delta}{\beta-\delta}}$$

Let $\varsigma:=\frac{\varphi(1-\beta)}{\underbrace{\delta(1-\beta)-\beta(1-\delta)}_{\geq 0}}\ \Rightarrow\ \varsigma\geq 0$ and let also $\tau:=\frac{\varphi\beta}{\underbrace{\delta(1-\beta)-\beta(1-\delta)}_{\geq 0}}\ \Rightarrow\ \tau\geq 0$

$$\Rightarrow x(k_t,p_t)=\varsigma k_t(\tilde{p}_t\theta)^{\frac{\delta-1}{\beta-\delta}}-\tau(\tilde{p}_t\theta)^{\frac{\delta}{\beta-\delta}} \tag{2.79}$$

And the temporary equilibrium output of the non-tradable good, n is given by:

$$n(k_t,p_t)=l_t^n f_n\left(k^n\left(\omega(p_t)\right)\right)$$

$$\Rightarrow n(k_t,p_t)=\frac{k_t-\frac{\delta}{1-\delta}(\tilde{p}_t\theta)^{\frac{1}{\beta-\delta}}}{\left(\frac{\beta}{1-\beta}-\frac{\delta}{1-\delta}\right)(\tilde{p}_t\theta)^{\frac{1}{\beta-\delta}}}\frac{\alpha}{\beta}\left(\frac{\beta}{1-\beta}(\tilde{p}_t\theta)^{\frac{1}{\beta-\delta}}\right)^{\beta} \tag{2.80}$$

$$\Rightarrow n(k_t,p_t)=\frac{1-\beta(1-\delta)\left(k_t(\tilde{p}_t\theta)^{-\frac{1}{\beta-\delta}}-\frac{\delta}{1-\delta}\right)}{\beta(1-\delta)-\delta(1-\beta)}\frac{\alpha}{\beta}\left(\frac{\beta}{1-\beta}(\tilde{p}_t\theta)^{\frac{1}{\beta-\delta}}\right)^{\beta}$$

$$\Rightarrow n(k_t,p_t)=\frac{1-\beta(1-\delta)\left(k_t(\tilde{p}_t\theta)^{-\frac{1}{\beta-\delta}}-\frac{\delta}{1-\delta}\right)}{\beta(1-\delta)-\delta(1-\beta)}\frac{\alpha}{\beta}\left(\frac{\beta}{1-\beta}(\tilde{p}_t\theta)^{\frac{1}{\beta-\delta}}\right)^{\beta}$$

$$\Rightarrow n(k_t,p_t)=\frac{\alpha}{\beta}\left(\frac{\beta}{1-\beta}\right)^{\beta}\frac{1-\beta(1-\delta)}{\beta(1-\delta)-\delta(1-\beta)}k_t(\tilde{p}_t\theta)^{\frac{\beta-1}{\beta-\delta}}$$

$$\frac{\alpha}{\beta}\left(\frac{\beta}{1-\beta}\right)^{\beta}\frac{\left(\delta(1-\beta)\right)}{\beta(1-\delta)-\delta(1-\beta)}(\tilde{p}_t\theta)^{\frac{\beta}{\beta-\delta}}$$

$$\Rightarrow n(k_t,p_t)=\frac{\alpha}{\beta}\left(\frac{\beta}{1-\beta}\right)^{\beta}\frac{1-\beta(1-\delta)}{\beta(1-\delta)-\delta(1-\beta)}k_t(\tilde{p}_t\theta)^{\frac{\beta-1}{\beta-\delta}}-$$

$$\frac{\alpha}{\beta}\left(\frac{\beta}{1-\beta}\right)^{\beta}\frac{\left(\delta(1-\beta)\right)}{\beta(1-\delta)-\delta(1-\beta)}(\tilde{p}_t\theta)^{\frac{\beta}{\beta-\delta}}$$

Let $\xi:=\frac{\alpha}{\beta}\left(\frac{\beta}{1-\beta}\right)^{\beta}\frac{1-\beta(1-\delta)}{\beta(1-\delta)-\delta(1-\beta}$ and $\chi:=\frac{\alpha}{\beta}\left(\frac{\beta}{1-\beta}\right)^{\beta}\frac{\left(\delta(1-\beta)\right)}{\beta(1-\delta)-\delta(1-\beta)}$

$$\Rightarrow n\left(k_t, p_t\right) = \xi k_t \left(\tilde{p}_t \theta\right)^{\frac{\beta-1}{\beta-\delta}} - \chi \left(\tilde{p}_t \theta\right)^{\frac{\beta}{\beta-\delta}} \tag{2.81}$$

2.3 Capital Account

The portfolio allocation problem facing the representative household is that he wants to allocate his income after consumption (that is his savings) between the two alternative investment choices. He wants to invest in domestic assets B_t and foreign assets B_t^*, i.e.

$$s_t = \left(B_t / e_t + B_t^* \right) \tag{2.82}$$

We assume that the consumer must hold both assets in positive amounts so that none of the asset demands is zero. This guarantees that the returns from the two investment opportunities are equal. The household's investments in domestic assets earn a return of i_t, while investment in the foreign assets earns a return equal to i_t^*. The returns from the two investment alternatives are assumed to be the same, so that the two assets are perfect substitutes. The net purchase of foreign assets by the residents (net capital outflows) is given by:

$$Inf_t = \left(1 - i_t^*\right) e_t B_t^* \tag{2.83}$$

Equation (2.83) shows that the residents of this country invest $e_t B_t^*$ in foreign bonds outside the country and out of this they get back $i_t^* e_t B_t^*$ as interest on their foreign investments. The net capital outflows Inf_t in period *t* therefore equals the amount invested out of country in period *t*, minus the interest from the foreign bonds repatriated back to the country. Capital outflows Inf_t is therefore a decreasing function of the foreign interest rates.

2.4 External Balance

According to Edwards (1989), the external balance occurs when the balance of payments sums up to zero. The balance of payments equation is given as:

$$\Delta R_t = BT\left(e_t\right) - Inf_t\left(i_t^*\right) = 0 \tag{2.84}$$

where $BT\left(e_t\right) = \tilde{p}_t . x_t\left(\tilde{p}_t\right) - e_t . m_t^d\left(e_t\right)$ is the balance of trade relation, e_t is the exchange rate, $Inf_t(.)$ is the capital account relation, and i_t^* is the foreign interest rate. The balance of trade is the difference between the values of goods and services that a country exports and the value of the goods and services that it imports. If a country's exports exceed its imports, it has a trade surplus and the trade balance is positive. If imports exceed exports, the country has a trade deficit and its trade balance is negative. The following conditions therefore sum up the temporary equilibrium conditions.

2.5 Temporary Equilibrium Conditions

2.5.1 Factor market equilibrium conditions

$$K_t^n + K_t^x = K_t \Longrightarrow k_t^n l_t^n + k_t^x l_t^x = k_t \tag{2.85}$$

$$L_t^n + L_t^x = L_t \Longrightarrow l_t^n + l_t^x = 1 \tag{2.86}$$

$$k_{t+1} = l_t^x f_x\left(k_t^x\right) \tag{2.87}$$

2.5.2 Non-tradable goods market equilibrium conditions

$$n_t^{\,d} + g_t^{\,n} = n_t^{\,s} \tag{2.88}$$

2.5.3 Fiscal balance (Government sector)

$$\left(g_t^{\,n} + e_t g_t^{\,m}\right) + \left(1 - i_{t-1}\right) B_{t-1} = \left(tax\right) w_t + B_t \tag{2.90}$$

2.5.4 Tradable goods conditions (Trade balance condition)

$$\Delta R_t = BT_t\left(e_t\right) - Inf_t\left(i_t^*\right) = 0 \tag{2.91}$$

2.6 Internal Temporary Equilibrium

The non-traded goods market equilibrium implies that household demand of the non-traded goods plus the government demand of the non-traded goods equals the supply of the non-traded goods by the firms.

i.e. $n^d + g^n = n^s$

$$n^d + g^n = \rho\left(\left(1 - tax\right) w_t - e_t s_t\right) + g^n$$

$$n^s = \xi k_t \left(\tilde{p}_t \theta\right)^{\frac{\beta-1}{\beta-\delta}} - \chi\left(\tilde{p}_t \theta\right)^{\frac{\beta}{\beta-\delta}}$$

$$n^d + g^n = n^s \Rightarrow \rho\left(\left(1 - tax\right) w_t - e_t s_t\right) + g_t^{\,n} = \xi k_t \left(\tilde{p}_t \theta\right)^{\frac{\beta-1}{\beta-\delta}} - \chi\left(\tilde{p}_t \theta\right)^{\frac{\beta}{\beta-\delta}}$$

$$\Rightarrow \chi\left(\tilde{p}_t \theta\right)^{\frac{\beta}{\beta-\delta}} - \xi k_t \left(\tilde{p}_t \theta\right)^{\frac{\beta-1}{\beta-\delta}} + \rho\left(\left(1 - tax\right) w_t - e_t s_t\right) + g_t^{\,n} = 0 \tag{2.92}$$

As noted earlier in section 2.2.3, Montiel (1999) refers to the e_t that solves equation (2.92) as the short-run equilibrium real exchange rate. The short run real exchange rate is therefore given as:

$$\Rightarrow \tilde{e}_t = \frac{\chi(\tilde{p}_t\theta)^{\frac{\beta}{\beta-\delta}} - \xi k_t(\tilde{p}_t\theta)^{\frac{\beta-1}{\beta-\delta}} + \rho(1-tax)w_t + g_t^{\,n}}{\rho s_t}$$

$$\tilde{e}_t = \frac{(1-tax)w_t}{s_t} - \left(\frac{\chi(\tilde{p}_t\theta)^{\frac{\beta}{\beta-\delta}} - \xi k_t(\tilde{p}_t\theta)^{\frac{\beta-1}{\beta-\delta}} + g_t^{\,n}}{s_t\rho}\right)$$

where $\tilde{e}_t$ denotes the short run real exchange rates.

2.7 Balance of Payments Equilibrium

The balance of payments equation above can then be written as:

$\Delta R_t = BT(e_t) - Inf(i_t^*) = 0$

but $BT(e_t) = \tilde{p}_t.x_t(\tilde{p}_t) - e_t.m_t^d(e_t)$ where $m_t^{\,d} = m_t + g_t^{\,m}$

$$\tilde{p}_t.\varsigma k_t(\tilde{p}_t\theta)^{\frac{\delta-1}{\beta-\delta}} - \tilde{p}_t.\tau(\tilde{p}_t\theta)^{\frac{\delta}{\beta-\delta}} = e_t.(1-\rho)\frac{(1-tax)w_t - e_t s_t}{e_t} - e_t.g_t^{\,m} - (1-i_t^*)e_t B_t^* \quad (2.93)$$

2.8 Internal-External Equilibrium

The internal equilibrium is given as:

$$\Rightarrow \chi(\tilde{p}_t\theta)^{\frac{\beta}{\beta-\delta}} - \xi k_t(\tilde{p}_t\theta)^{\frac{\beta-1}{\beta-\delta}} + \rho((1-tax)w_t - e_t s_t) + g_t^{\,n} = 0$$

The external balance is given as:

$$\tilde{p}.\varsigma k_t(\tilde{p}_t\theta)^{\frac{\delta-1}{\beta-\delta}} - \tilde{p}.\tau(\tilde{p}_t\theta)^{\frac{\delta}{\beta-\delta}} - e_t.(1-\rho)\frac{(1-tax)w_t - e_t s_t}{e_t} - e_t.g_t^{\,m} - (1-i_t^*)e_t B_t^* = 0$$

To obtain the internal-external equilibrium, we solve simultaneously for e_t, thus:

$$\chi(\tilde{p}_t\theta)^{\frac{\beta}{\beta-\delta}} - \xi k_t(\tilde{p}_t\theta)^{\frac{\beta-1}{\beta-\delta}} + \rho((1-tax)w_t - e_t s_t) + g_t^{\,n} = 0 =$$
$$\tilde{p}_t.\varsigma k_t(\tilde{p}_t\theta)^{\frac{\delta-1}{\beta-\delta}} - \tilde{p}_t.\tau(\tilde{p}_t\theta)^{\frac{\delta}{\beta-\delta}} - e_t.(1-\rho)\frac{(1-tax)w_t - e_t s_t}{e_t} - e_t.g_t^{\,m} - (1-i_t^*)e_t B_t^* \quad (2.94)$$

$$\chi(\tilde{p}_t\theta)^{\frac{\beta}{\beta-\delta}} + \tilde{p}_t.\tau(\tilde{p}_t\theta)^{\frac{\delta}{\beta-\delta}} - \tilde{p}_t.\varsigma k_t(\tilde{p}_t\theta)^{\frac{\delta-1}{\beta-\delta}} - \xi k_t(\tilde{p}_t\theta)^{\frac{\beta-1}{\beta-\delta}} + \rho(1-tax)w_t + (1-\rho)(1-tax)w_t + g_t^{\,n} =$$
$$\left[\rho e_t s_t - (1-\rho)e_t s_t - e_t.g_t^{\,m} - (1-i_t^*)e_t B_t^*\right]$$

$$\chi(\tilde{p}_t\theta)^{\frac{\beta}{\beta-\delta}} + \tilde{p}_t.\tau(\tilde{p}_t\theta)^{\frac{\delta}{\beta-\delta}} - \tilde{p}_t.\varsigma k_t(\tilde{p}_t\theta)^{\frac{\delta-1}{\beta-\delta}} - \xi k_t(\tilde{p}_t\theta)^{\frac{\beta-1}{\beta-\delta}} + \rho(1-tax)w_t + (1-\rho)(1-tax)w_t + g_t^{\,n} =$$
$$\left[\rho e_t s_t - (1-\rho)e_t s_t - e_t.g_t^{\,m} - (1-i_t^*)e_t B_t^*\right]$$

$$\chi(\tilde{p}_t\theta)^{\frac{\beta}{\beta-\delta}} + \tilde{p}_t.\tau(\tilde{p}_t\theta)^{\frac{\delta}{\beta-\delta}} - \tilde{p}_t.\varsigma k_t(\tilde{p}_t\theta)^{\frac{\delta-1}{\beta-\delta}} - \xi k_t(\tilde{p}_t\theta)^{\frac{\beta-1}{\beta-\delta}} + (1-tax)w_t + g_t^{\,n} =$$
$$e_t\left[\rho s_t - (1-\rho)s_t - g_t^{\,m} - (1-i_t^*)B_t^*\right] \quad (2.95)$$

$$\overline{e}_t = \frac{\left[\tilde{p}_t.\tau\left(\tilde{p}_t\theta\right)^{\frac{\delta}{\beta-\delta}} - \tilde{p}_t.\varsigma k_t\left(\tilde{p}_t\theta\right)^{\frac{\delta-1}{\beta-\delta}}\right]\left[\chi\left(\tilde{p}_t\theta\right)^{\frac{\beta}{\beta-\delta}} + -\xi k_t\left(\tilde{p}_t\theta\right)^{\frac{\beta-1}{\beta-\delta}}\right] + (1-tax)w_t + g_t^{\,n}}{\left[\rho s_t - (1-\rho)s_t - g_t^{\,m} - \left(1-i_t^*\right)B_t^*\right]} \quad (2.96)$$

where $\overline{e}_t$ is the long-run equilibrium real exchange rate. Equation (2.96) gives the equilibrium real exchange rate conditional on a vector of permanent values for the fundamentals.

2.9 Real Shocks and the Equilibrium Real Exchange Rates

This section determines how the equilibrium real exchange rates will behave as a result of changes from the different fundamentals.

2.9.1 Effects of government spending shocks on the real exchange rates

$$\partial\overline{e}_t \Big/ \partial g_t^m = \frac{\chi\left(\tilde{p}_t\theta\right)^{\frac{\beta}{\beta-\delta}} + \tilde{p}_t.\tau\left(\tilde{p}_t\theta\right)^{\frac{\delta}{\beta-\delta}} - \tilde{p}_t.\varsigma k_t\left(\tilde{p}_t\theta\right)^{\frac{\delta-1}{\beta-\delta}} - \xi k_t\left(\tilde{p}_t\theta\right)^{\frac{\beta-1}{\beta-\delta}} + (1-tax)w_t + g_t^{\,n}}{\left[\rho s_t - (1-\rho)s_t - g_t^{\,m} - \left(1-i_t^*\right)B_t^*\right]^2} \geq 0 \quad (2.97)$$

The numerator in equation (2.97) above is positive because of the demand of the non-traded good $\chi\left(\tilde{p}_t\theta\right)^{\frac{\beta}{\beta-\delta}} - \xi k_t\left(\tilde{p}_t\theta\right)^{\frac{\beta-1}{\beta-\delta}} \geq 0$, government expenditures $g_t^{\,n} \geq 0$ and disposable income $(1-tax)w_t \geq 0$. The denominator is also positive since it is squared. Therefore , $\partial\overline{e}_t \big/ \partial g_t^m \geq 0$, i.e. the change in long run real exchange rate as a result of an increase in government expenditure on traded goods is positive. This means that the long-run real exchange rate depreciates as a result of an increase in government spending on the traded goods. Montiel (1999) argues that an increase in government spending creates an incipient trade deficit, which requires a real depreciation in order to maintain external balance.

2.9.2 Effects of international interest rates shocks on the real exchange rates

$$\partial\overline{e}_t \Big/ \partial i_t^* = \frac{-B_t^*\left[\chi\left(\tilde{p}_t\theta\right)^{\frac{\beta}{\beta-\delta}} + \tilde{p}_t.\tau\left(\tilde{p}_t\theta\right)^{\frac{\delta}{\beta-\delta}} - \tilde{p}_t.\varsigma k_t\left(\tilde{p}_t\theta\right)^{\frac{\delta-1}{\beta-\delta}} - \xi k_t\left(\tilde{p}_t\theta\right)^{\frac{\beta-1}{\beta-\delta}} + (1-tax)w_t + g_t^{\,n}\right]}{\left[\rho s_t - (1-\rho)s_t - g_t^{\,m} - \left(1-i_t^*\right)B_t^*\right]^2} \geq 0$$

The terms in brackets in the numerator are all positive, i.e. $g_t^{\,n} \geq 0$ and $(1-tax)w_t \geq 0$. The numerator is therefore negative because of the negative sign outside the bracket. The denominator is positive since it is squared. Therefore, $\partial\overline{e}_t \big/ \partial B_t^* \geq 0$, meaning the real exchange rate appreciates as a result of an increase in foreign interest rates. The results are again consistent with the findings of Montiel (1999) which shows that a change in the world interest rates leads to a change in the real exchange rates, in the opposite direction. Therefore, an increase in the world interest rates is expected to lead to a reduction in the index of the real exchange rates which signifies an appreciation of the real exchange rates. A decrease in the world interest rates will, on the other hand, depreciate the real exchange rates.

2.9.3 Effects of capital outflow shocks on the real exchange rates

$$\partial \overline{e}_t \big/ \partial B_t^* = \frac{\left(1-i_t^*\right)\left[\chi\left(\tilde{p}_t\theta\right)^{\frac{\beta}{\beta-\delta}} + \tilde{p}_t.\tau\left(\tilde{p}_t\theta\right)^{\frac{\delta}{\beta-\delta}} - \tilde{p}_t.\varsigma k_t\left(\tilde{p}_t\theta\right)^{\frac{\delta-1}{\beta-\delta}} - \xi k_t\left(\tilde{p}_t\theta\right)^{\frac{\beta-1}{\beta-\delta}} + \left(1-tax\right)w_t + g_t^{\,n}\right]}{\left[\rho s_t - \left(1-\rho\right)s_t - g_t^{\,m} - \left(1-i_t^*\right)B_t^*\right]^2} \geq 0$$

The terms in the numerator are all positive and the term in the denominator is also positive. Therefore $\partial \overline{e}_t \big/ \partial i_t^* \leq 0$. Since B_t^* is the amount of resident's investment on foreign bonds, it represents capital outflows. The results imply that real exchange rates will depreciate as a result of an increase in outflow of capital out of the country. These findings are consistent with the findings of Montiel (1999), which argue that an increase in receipts of transfer incomes from abroad (capital inflows) shifts the external balance to the right, which permits the expansion of consumption. This leads to an appreciation of the real exchange rates. Capital outflows, on the other hand, are expected to depreciate the real exchange rate through the same channel.

3. Summary and Conclusions

This study develops a theoretical general equilibrium model for determining the real exchange rates from a representative consumer consuming two goods: the non-tradable good and the importable good; a representative firm producing two goods: the non-tradable good and the exportable good in a two-sector framework; and the government sector which consumes both the non-tradable good and the importable good. The three agents interact in an internal-external equilibrium framework, with the internal equilibrium determined by the demand and supply of the non-tradable good and the external equilibrium determined by the balance of payments equilibrium.

Analyzing the impacts of different fundamentals as obtained in the developed equilibrium real exchange rate equation shows that the long run real exchange rate depreciates as a result of an increase in government spending on the traded goods. This is the same result obtained by Montiel (1999). The comparative statistics further show that the real exchange rates appreciates as a result of an increase in the foreign interest rates. The results here are again consistent with the findings of Montiel (1999), which shows that a change in the world interest rates leads to a change in the real exchange rates in the opposite direction. In addition, the real exchange rates are found to depreciate as a result of an increase in outflow of capitals. These findings are again consistent with the findings of Montiel (1999).

In general, this study adds to the existing knowledge of the determination of the long-run real exchange rates by suggesting that the real sector variables, including capital labour ratio, are important determinants of the long-run real exchange rates.

References

Edwards S. (1989), Real Exchange Rates, Devaluations and Adjustments, Cambridge, Mass, MIT Press.

Galor O. (1992) "A Two-Sector Overlapping-Generations Model: A Global Characterization of the Dynamical System", *Econometrica*, Vol. 60, No. 6.

McDonald R. (2000) "Concepts to Calculate Equilibrium Exchange Rates: An Overview", Deutsche Bundesbank, Discussion Paper No. 3/00.

Montiel, P. (1999), "Determinants of the Long-Run Real Exchange Rate", in Lawrence E. Hinkle and Peter J. Montiel (eds), *Exchange Rate Misalignments: Concepts and Measurements for Developing countries*, New York: Oxford University Press.

Schmitz, O. (2007), "Dynamic Behaviour in a Two-Sector Neoclassical Growth Model with Differential Savings", PhD Dissertation, Bielefeld University Library, Germany.

Teodorescu, M. (2008), "Autarky versus Free Trade: An Endogenous Growth Approach", Unpublished PhD Thesis, University of Bielefeld, Germany.

Kenya Institute for Public Policy Research and Analysis
Bishops Garden Towers, Bishops Road
PO Box 56445, Nairobi, Kenya
tel: +254 20 2719933/4, 2714714/5, 2721654, 2721110
fax: +254 20 2719951
email: admin@kippra.or.ke
website: http://www.kippra.org

ISBN 9966 777 69 5